STRUCTURAL DAMAGE DETECTION USING THE MODAL CORRELATION COEFFICIENT (MCC)

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ABSTRACT. *Since its introduction by Allemang and Brown in 1982 [1], the Modal Assurance Criterion (MAC) has been the main quantitative measure of correlation for mode shapes because it is a simple yet capable means of numerically expressing the similarity or difference between two mode shapes. In 1996, the Modal Correlation Coefficient (MCC) which is a modification of the MAC was introduced [2]. The new measure was devised for damage detection applications, structural health monitoring, and diagnostics. In this paper, the properties of MCC are discussed and its sensitivity to spatial resolution is studied. Several examples are given to illustrate its capabilities and to show how ii maintains the simplicity and orthogonality of the standard MAC. Also, the paper shows how damage can be detected using MCC even if the accompanied magnitude changes are minimal.*

NOMENCLATURE

$\mathbf{c}^x$, $\mathbf{c}^y$	vectors of the slope change in mode shapes **x** and y
d	distance vector where d_i is the distance between measurement points *i* and *i+ I*
x , **y**	mode shape vectors
i	dummy index
$k(\mathbf{x},\mathbf{y})$	kink factor for mode shapes **x** and y
$M(\mathbf{x},\mathbf{y})$	Modal Assurance Criterion for mode shapes **x** and y
$C(\mathbf{x},\mathbf{y})$	Modal Correlation Coefficient for mode shapes x and y
n	number of points in mode shape vectors
s	total span length of the tested structure

1. INTRODUCTION

Since its introduction in 1982, the Modal Assurance Criterion (MAC) has been the main quantitative measure of correlation for mode shapes. [1] It was adopted by workers in the field because it was a simple yet capable means of numerically expressing the similarity or difference between mode shapes.

The MAC of mode shapes **x** and y is

$$M(\mathbf{x},\mathbf{y}) = \left|\mathbf{x}^T.\mathbf{y}\right|^2 / (\mathbf{x}^T.\mathbf{x}.\mathbf{y}^T.\mathbf{y}) \qquad (1)$$

It is a real coefficient that takes values between 0 and 1. A value of one indicates identical mode shapes. An important property of this coefficient is that it accounts for the orthogonality of mode shapes. In other words, if

$$y = -\mathbf{x} \quad (2)$$

then

$$M(\mathbf{x}, \mathbf{y}) = 1 \quad (3)$$

Since the MAC is a magnitude-based coefficient, it has one shortcoming. It is not suited for detecting shape differences in mode shapes if the corresponding magnitude changes are minimal. This deficiency creates a problem in many damage detection applications in which the objective is to detect kinks in mode shapes.

For example, the formation of a severe crack in the middle of a simply supported beam can change its first mode shape from a smooth curve to one with a kink and relatively straight sides as shown in Figure 1. From a health monitoring stand point, this change can be indicative of a catastrophic defect. Nevertheless, the corresponding MAC for the two mode shapes shown in the figure is 0.985 which usually indicates almost identical mode shapes. Therefore, in this case, the severity of the difference is not well expressed using MAC.

From the above example, the need for a measure that accounts for kink formation is evident. The new measure needs to satisfy the following minimal conditions:

a. It should be sensitive to slope discontinuities even if magnitude changes are negligible as in the example of Figure 1.

b. It should maintain its sensitivity to kink formations even if they exist in a small part of a relatively large structure such as the four-span continuous beam shown in Figure 2.

c. It should maintain the orthogonality property of MAC as expressed in equations (2) and (3) above and shown in Figure 3.

d. It should be as sensitive as MAC to magnitude changes that are not accompanied by kinks such as the one in Figure 4.

2. THEORY

Kinks are points at which slope discontinuities exist. A kink at a point manifests itself on the first derivative curve as a sudden change from a large positive value to a large negative value or vice versa. To detect a kink, the value of the incidental change of slope can be used as an indicator. If a single kink exists in a mode shape, the slope change at the location of the kink is expected to be a lot larger in magnitude than at any other location on the structure.

Therefore, the Modal Correlation Coefficient for $\mathbf{x}$ and y is defined as, [2]

$$C(\mathbf{x}, \mathbf{y}) = M(\mathbf{x}, \mathbf{y}) * k(\mathbf{x}, \mathbf{y}) \quad (4)$$

where k (x , y) is the kink factor which is a real coefficient that compares the existence of kinks in $\mathbf{x}$ and y and is defined using the following equations,

$$k(\mathbf{x}, \mathbf{y}) = \min(\|c^x\|_\infty, \|c^y\|_\infty) / \max(\|c^x\|_\infty, \|c^y\|_\infty) \quad (5)$$

$$\|c^x\|_\infty = \max(|c^x_2|, |c^x_3|, \ldots, |c^x_{n-2}|, |c^x_{n-1}|) \quad (6)$$

$$\|c^y\|_\infty = \max(|c^y_2|, |c^y_3|, \ldots, |c^y_{n-2}|, |c^y_{n-1}|) \quad (7)$$

$$c^x_i = (x_{i+1} - x_i)/d_i - (x_i - x_{i-1})/d_{i-1} \quad (8)$$

$$c^y_i = (y_{i+1} - y_i)/d_i - (y_i - y_{i-1})/d_{i-1} \quad (9)$$

$$\mathbf{d} = \{d_1, d_2 . d_3, \ldots, d_i, \ldots, d_{n-2}, d_{n-1}\} \quad (10)$$

where

n is the number of measurement points, and

d_i is the incremental distance between the points at which measurements i and $i+1$ are obtained such that the summation of all distances in the distance vector is equal to the total span length of the measured structure, s, or

$$\sum_{1}^{n-1} d_i = s \tag{11}$$

Therefore, MAC can be considered as a special case of MCC to which it reduces when the smoothness of the two mode shapes are identical (k =1.0). The following two important properties follow:

$$1 \geq k \geq 0 \tag{12}$$

and

$$1 \geq C \geq 0 \tag{13}$$

The effectiveness of MCC is dependent on the smoothness of mode shapes. Since experimental mode shapes are obtained using vibration measurements at discrete points, the smoothness of mode shapes is a function of the spatial distance between these points. As measurement points get closer, mode shapes look smoother. Therefore, the MCC is more sensitive to kinks when measurement points are closely spaced.

To illustrate the influence of the number of points on the result, we used the two mode shapes of the simply supported single-span beam in Figure 1 with different number of points. The change in M and C values are shown in Figure 5. As the figure indicates, M stays close to unity regardless of the number of points. But, C decreases as the number of points increases and therefore becomes more sensitive to kinks when measurement points are numerous.

3. EXAMPLES

To verify that MCC satisties all four conditions set forth in the introduction, we used it to compare the mode shapes in Figures 1 through 4. The original mode shapes in all figures were sine functions. All spans were divided into 40 equal distances and, therefore, measurements were assumed at 41 equally spaced points. This number of divisions was arbitrarily chosen to provide relatively smooth mode shapes. The spans in which kinks exist were modeled using straight lines The corresponding values of M, k, and Care given in Table 1.

From the table, the kinks in Figures 1 and 2 that were undetectable using M are detectable using C. The corresponding values of k were 0.0617 and 0.2447, respectively. Note that the first number is a lot smaller than the second even though the number of measurement points are the same. This change occurred because that same number of points was distributed over four spans rather than over a single span.

As for the mode shapes of Figures 3 and 4, the value of k was unity in both cases and, therefore, did not alter the original value of M. These results satisfy the four conditions of the introduction.

4. CONCLUSIONS

In this paper, the Modal Correlation Coefficient (MCC) is evaluated with special emphasis on its damage detection. It was found that the MCC is sensitive to slope discontinuities (kinks) even if magnitude changes are negligible which is usually the case when comparing corresponding damaged and undamaged mode shapes. This sensitivity to kinks is maintained even if they exist in a small part of a relatively large structure. Also, the MCC maintains the desired properties of the Modal Assurance Criterion (MAC), namely its simplicity, orthogonality, and its sensitivity to magnitude changes that are not accompanied by kinks. This new measure is not meant to replace the standard MAC, but rather to compliment in damage detection and health monitoring application. The MCC should not be used in applications where kinks are common or when measurement points are too far apart

5. ACKNOWLEDGMENTS

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6. REFERENCES

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	Fig.1	Fig.2	Fig.3	Fig.4
M	0.9853	0.9899	1.0000	0.4878
k	0.0617	0.2447	1.0000	1.0000
C	0.0607	0.2422	1.0000	0.4878

Table 1 A comparison of *M*, *k*, and C for the mode shapes in Figures 1 through 4

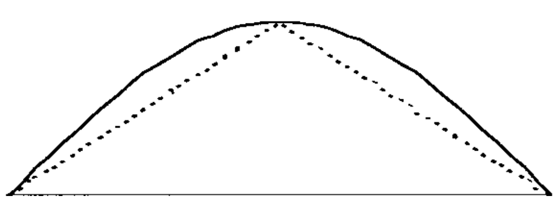

Fig. 1 Two single span modes

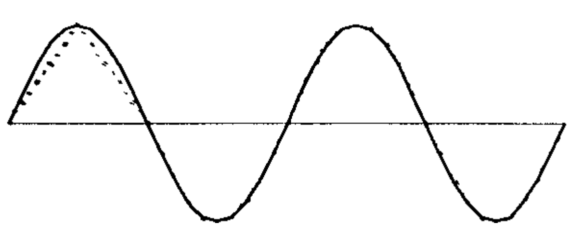

Fig.2 Two multiple span modes

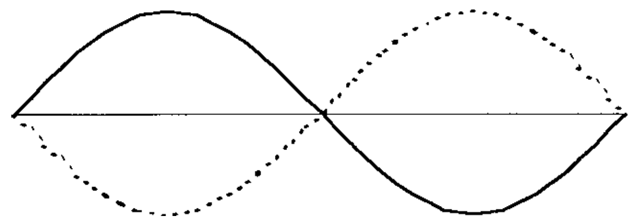

Fig.3 Two orthogonal modes

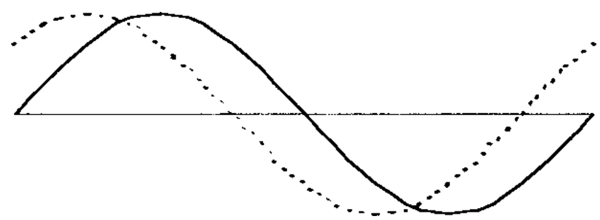

Fig.4 Two modes with no kinks

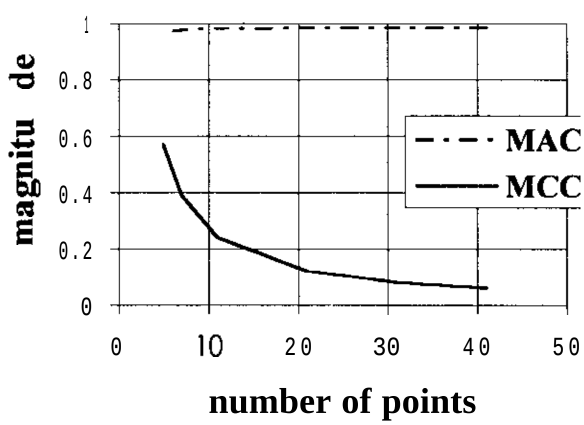

Fig.5 The sensitivity of MAC and MCC to the number of measurement points